# TOMOGRAPHY SIMULATION AND RECONSTRUCTION TOOLS APPLIED IN THE EVALUATION OF PARAMETERS AND TECHNIQUES

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*Abstract* -**In this paper we introduce a tomography platform consisting of a new set of programs written in ANSI C++ to help researchers to develop and to evaluate tomography simulation/reconstruction based on the Direct Fourier Method. We applied these tools in the evaluation of tomographic techniques. We describe and compare two techniques to overcome the inferior image quality due to the inherent artifacts of the Direct Fourier Method: the increase of the zero-padding factor and the resampling grid density factor.**
***Keywords -*** **Tomography, fan-beam, sinogram, reconstruction, Direct Fourier Method**

## I. INTRODUCTION

Tomographic techniques are used in radiology and in many branches of science and technology for imaging 2D cross sections of 3D objects [1]. There are many methods to perform data acquisition.

The Direct Fourier Method (DF Method) and the Filtered Backprojection Method (FB Method) are two of the most well-known heuristics for tomographic reconstruction. The time complexity of these two methods are O(N2.logN) and O(N3) respectively. The disadvantage in speed presented by the Filtered Backprojection Method is compensated by an alleged superiority in picture quality (less artifacts) [1].

We designed and implemented a tomography platform consisting of a new set of tools for tomographic image simulation and reconstruction based on the Direct Fourier Method.

This paper presents these tools and applies them in the proposal of two tomographic techniques to improve the inferior image quality of the DF Method as compared to the FB Method. To overcome the inherent artifacts of the DF Method we propose and evaluate: (1) the increase of the zero-padding factor and (2) the resampling grid density factor.

## II. METHODOLOGY

### The simulation tool

This tool was developed using the third-generation scanner. The system involves rotation-only of a fan beam, where both the source and the detector are rotated around a common center within the object.

As shown in Fig. 1, a set of projections (density function) $g(\theta_i,\rho)$ is obtained from each object slice for all rotation angles $\theta_i$. A projection consists of a collection of line integrals of the object attenuation coefficients $\mu(x, y)$ corresponding to the various rays coming from the source[5].

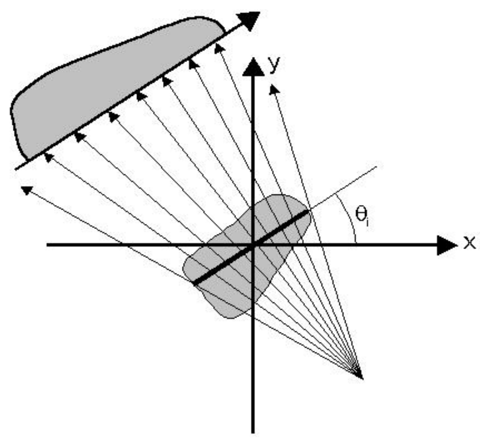


Fig 1. **Components of a projection**

Thus, the equation of the intensity at the detector is given by[2]:

$$I_d(x_d, y_d) = I_i(x_d, y_d)\cdot e^{-\ \mu(x,y)\cdot dr} \tag{1}$$

Where:
$I_d(X_d, Y_d)$ is the beam intensity at the detector;
$I_i(x_d, y_d)$ is the incident x-ray beam intensity in the absence of any attenuating object;
$\mu(x, y)$ is the attenuation coefficient of the object in the position x, y;
$d_r$ is the elementary object under study.

The simulation software provides tools for calculating projection data in a Personal Computer. We can see in Equation (1) that the intensity at a point in the detector is proportional to the number of photons per unit area. This intensity is attenuated by every element of the imaged object with attenuation coefficient $\mu(x, y)$. Intensity is energy per unit area. The DF Method explains that it is possible to reconstruct an object using "energy ratios" [1]. We can rewrite Equation (1) as an energy ratio:

$$\frac{I_d}{I_i} = e^{-\ \mu(x,y)\cdot dr} \tag{2}$$

This is a well-know equation which describes how x-rays are attenuated when traveling through an object [1]. In fact, in a simulation, we need only three different variables: (1) the distance between the source and the detector, (2) the

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attenuation coefficient of the object, (3) and the depth of the object in the direction of the ray.

The Fourier Method assumes that all projections need to be parallel. In practice, a real tomograph utilizes fan-beam projections, where, for high speed data acquisition, a single ray source is placed at a fixed position relative to the detector array (Fig. 2). To correct this geometry problem, we need to reassemble the projection data from various fan-beam projections to get parallel data: the rebinning phase.

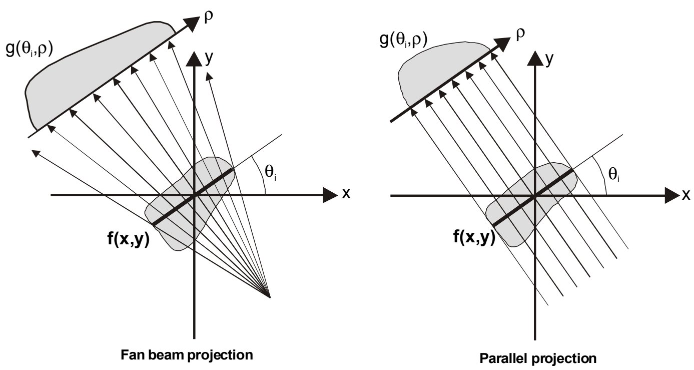



Fig 2**. The Rebinning.**

The simulation of the human body, in projection radiology, is called a phantom. In our simulation tool the phantom is described in two dimensions as a collection of ellipses with different density functions. The use of ellipses to describe tissues is very convenient because we can express them as analytical functions. In our experiments we have used the “Shepp-Logan Head Phantom”, one standard phantom in the literature (Fig. 3) [2].

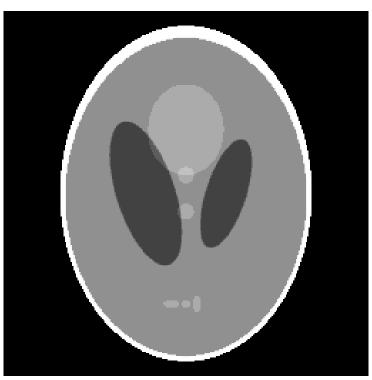


Fig. 3. **The “Shepp-Logan Head Phantom”.**

The output data of the simulation tool is piled up in a sinogram file. A sinogram is the collection of parallel projections of the object taken at equidistant angles, and forming a map of the projection data.

In the simulation phase, the user is capable of choosing the phantom, the geometry, the number of sensors, the width of the collimators, the number of projections and the relative position of all system components (object, source, start angles, etc.).

The Reconstruction tool

The Direct Fourier Method is a direct application of the Fourier Slice Theorem. This theorem is illustrated in Fig. 4 and is stated as follows:

“The Fourier transform $G(\theta_i,\rho)$ of a parallel projection of an image f(x,y) taken at angle $\theta_i$ is found in the two-dimensional transform F(u,v) on a line subtending the angle $\theta_i$ with the u-axis.” (Fig. 4) [2].

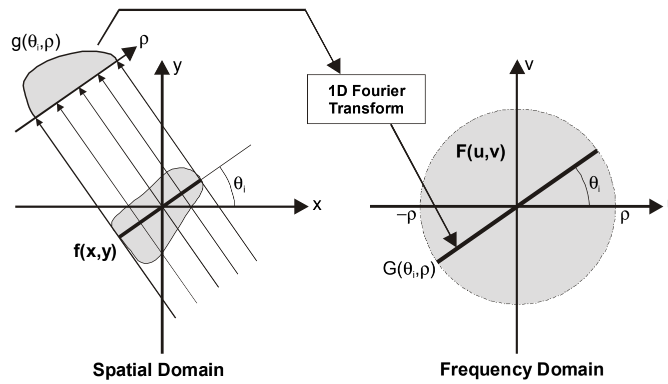



Fig 4. **The Fourier Slice Theorem.**

The Fourier Slice Theorem indicates that by taking the projections of an object function at angles $\theta_1$, $\theta_2$, ..., $\theta_n$ and Fourier transforming each of these projections, we can determine the values of F(u, v) radially. If an infinite number of projections is taken, then F(u, v) will be known at all points in the uv-plane, and the object function f(x, y) can be recovered by using the inverse 2D Fourier transform.

In practice only a finite number of projections of an object can be acquired, each projection consisting of a finite number of points. Therefore primarily, F(u, v) is only known at discrete points along the radial lines. (fig. 5a)[4].

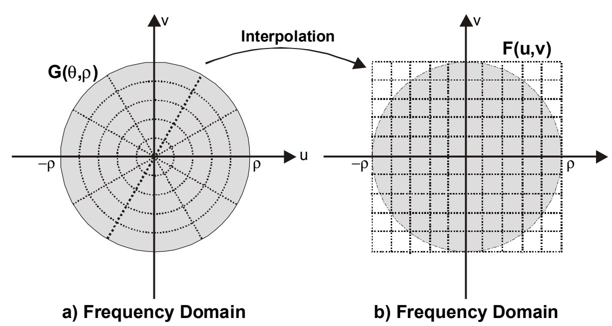



Fig. 5**. Interpolation in frequency domain.**

To compute the inverse Fourier transform (using a FFT algorithm) we must determine the values on a square grid

(Fig. 5b) using an interpolation method. A summary of Direct Fourier Method is shown in Fig. 6.

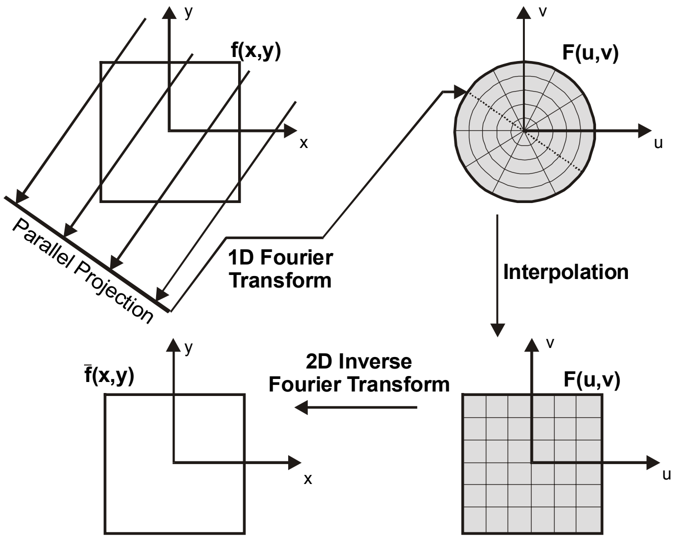



Fig 6. **Overview of Direct Fourier Method.**

In spite of the potential speed advantage of the DF Method as compared to the FB Method, it has not been used in practice because of its inferior image quality. There are two main sources of artifacts: the interpolation procedure and the circular convolution (inherent ramp filtering)[5].

To reduce the imperfect interpolation artifacts we can either use a more sophisticated interpolation procedure or we can append zeros in the Fourier domain (zero padding). We chose the zero-padding procedure which improves the space resolution. This was implemented by padding (($n_z$-1) N) zeros in the Fourier domain. The zero-padding factor ($n_z$) increases the number of data points per projection from N to $n_z$N[4].

One remedy for circular convolution in the signal domain is oversampling the F(u,v) function. The resampled grid density factor ($n_g$) increases the number of data points in the Cartesian Fourier Domain from $N^2$ to $(n_g N)^2$ [7].

In all our experiments we used the same sinogram with 256 sensors and 300 projections. Only two parameters have been varied during the reconstruction: the zero-padding factor ($n_z$) and the resampled grid density factor ($n_g$).

The measurements of how accurately the reconstruction has been performed are point-wise comparisons between the digitized picture of the object (the phantom) and the reconstructed image. To measure output image quality we used the error figures Relative Error and Absolute Error, defined as follows:

1) The Relative Error:

$$RE = \frac{\sum_y \sum_x \left| frec(x,y) - fobj(x,y) \right|}{\sum_y \sum_x \left| fobj(x,y) \right|} \tag{3}$$

2) The Absolute Error:

$$AE = \frac{1}{N^2} \cdot \sqrt{\sum_y \sum_x \left( frec(x,y) - fobj(x,y) \right)^2} \tag{4}$$

Where:
frec = reconstructed image.
Fobj = digitized picture of “phantom”.
N = size of square image.

## III. Results

Fig 7 shows the Relative Error variation versus the zero-padding factor ($n_z$) and the grid density factor ($n_g$).

Observe in Fig. 7 that the Relative Error decreases with the increase of the zero-padding factor, with best value for $n_z$ = 8. For greater zero-padding factors there is no significant improvement.

Concerning the grid density factor, notice in Fig 7 that the Relative Error decreases with the increase of the grid density factor ($n_g$). For $n_g$ > 4, however, the reconstruction time becomes too slow.

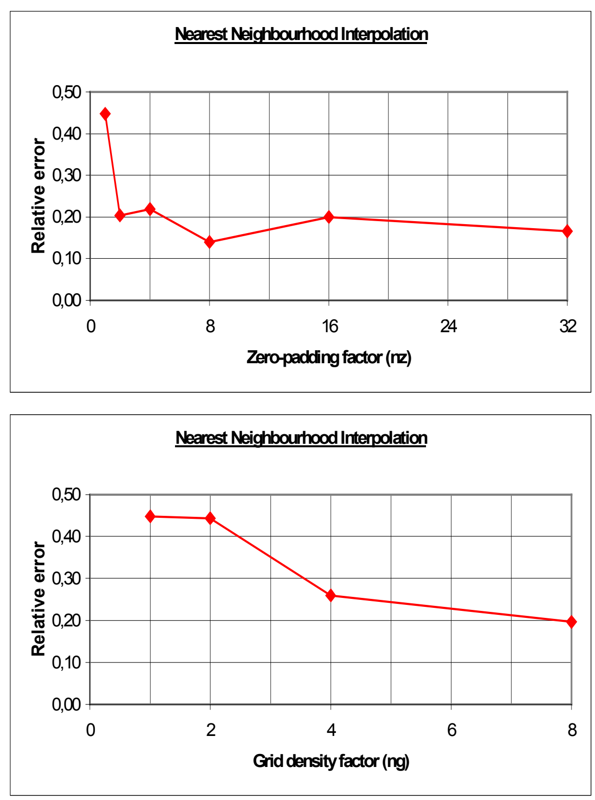
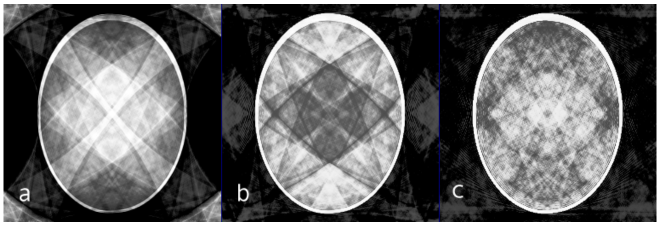
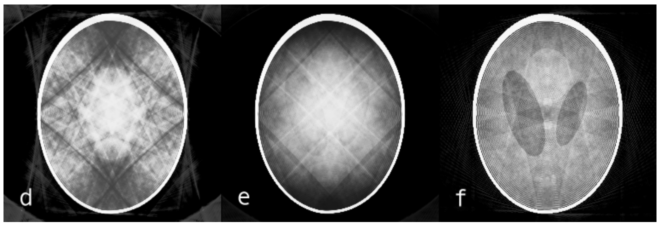



Fig. 7. **The Relative Error on reconstruction.**

Figure 8 shows a sequence of pictures reconstructed with the Direct Fourier Method using Nearest Neighborhood interpolation.



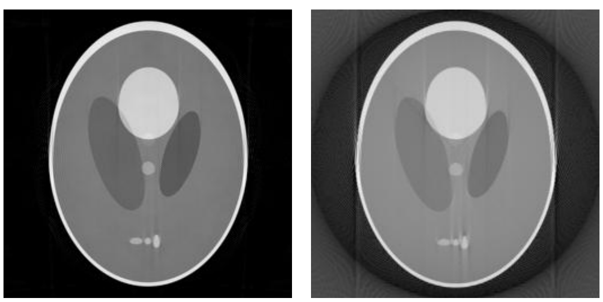

$n_z$=1, $n_g$=1 $n_z$=2, $n_g$=1 $n_z$=8, $n_g$=1




$n_z$=1, $n_g$=2 $n_z$=1, $n_g$=4 $n_z$=8, $n_g$=4

Fig. 8. **Reconstruction pictures.**

The most important experiment is shown in Fig. 9 which compare the DF Method and FB Method, respectively:



a b

Fig. 9. **Reconstruction with DF (a) and FB (b) Methods.**

For the same sinogram, our result tend to show that the DF Method is approximately as good as the FB Method. Admittedly, the extra filtering and interpolation increases the time complexity for the DF Method. Fig 10 shows the Relative Error plotted against the time to reconstruct an image. The upper graph in Fig 10 corresponds to the DF Method improved by the grid density factor. The lower graph in Fig. 10 corresponds to the DF Method improved by the zero padding factor. The reconstruction time using FB Method is 52s. Observe that for the same input reconstruction data (same sinogram) the DF Method improved by the zero padding factor 8 reconstructs the image in 2 seconds, while the FB Method reconstructs it in 52 s, for the same output image quality (about the same relative error of 0.1). The DF Method improved by the grid density factor reaches the same output image quality in 50 s, about the same time of the FB Method.

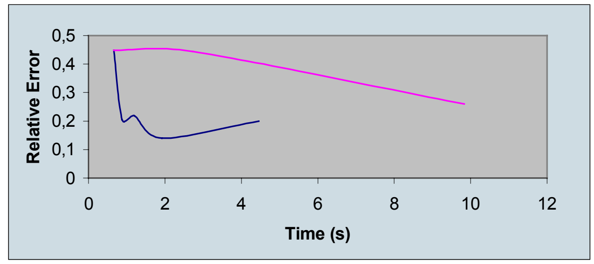



Fig. 10**. Time to reconstruct a same image**

## IV. Conclusion

We have successfully designed and implemented a tomography platform, consisting of a set of (1) tools to simulate the data generation in a tomograph and (2) a set of tools to reconstruct a 3D object based on its 2D projections. These tools, written in C++, should help investigators to develop and to evaluate tomographic techniques and parameters.

The tools were validated and used to study variations on reconstruction techniques. We proposed two variations of the DF Method: (1) by improvement of the grid density factor and (2) by improving the zero padding factor. We compared these two techniques and concluded that the zero padding factor technique is the best compromise in terms of processing time and output reconstructed image quality. We showed that the DF Method improved by the zero-padding factor could achieve the same reconstructed image quality of the FB Method in 1/26$^{th}$ of the time.

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